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Determination of the mean lifetime of solar features from photographic observations

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Summary. We consider the methods used for the quantitative determination of the mean lifetime of solar features from photographic observations and we derive a relation between the distribution of the lifetimes and the survival function. We examine the validity of the methods proposed by previous authors and we compare the results. We propose an accurate method for the measurement of the mean lifetime. Application of this method to previous measurements of the granule lifetimes gives a consistent value of $\approx$16 min for the mean lifetime with a standard deviation of $\approx$5 min.

Key words: Sun – granulation – data analysis

1. Introduction

The lifetime of various solar features (e.g. granules, bright points, bright and dark mottles etc.) is an important observational parameter, which is essential for the modeling of the associated physical phenomena. A rough estimate of the lifetime can be obtained through a qualitative description of the time evolution of a features (e.g. Bray and Loughhead, 1958) or by the use of some photographic techniques (e.g. "Moire Effect"; Macris, 1962). Quantitative measurements, give a more precise value of the mean lifetime. However, as a consequence of the different methods used and of the different definition of the term "mean lifetime", important discrepancies exist among values given by different authors. A typical case is the value of the mean lifetime of solar granules.

There are two commonly used techniques for the quantitative mean lifetime determination of solar features from photographic observations, based on the enumeration of the features under study.

The first (referred to as method 1 a in this paper) is based on the careful analysis of a time series of good-quality photographs of the same region. The first photograph of the series is considered as a "master" frame and all new emerging features are identified and followed from one photograph to another starting from the second photograph of the set (Bray and Loughhead, 1958). Then the mean lifetime (μ) of the features is given directly as the sum of N determinations of the lifetimes t_i, divided by the number of determinations:

$$\mu = (1/N) \sum_{i=1}^{N} t_i .$$

The same technique can be applied by using another enumeration method (method 1 b). A large sample of features is selected in the "master" frame (usually at the middle of the sequence) and they are traced back to formation and forward to disappearance (Macris, 1953). The advantage of this second enumeration method is that it is easier to trace every selected feature back to its formation instead of identifying every newborn feature.

Although very laborious this method, once used carefully, is the only one which permits to determine directly from the statistical sample the mean lifetime of the population, as well as other statistical parameters (standard deviation of lifetimes, type of distribution etc.). It gives also the possibility to examine if the features under study belong to the same statistical population, or the sample is composed of different populations.

In practice the applicability of this method is limited. A great disadvantage is that we need a long series of observations of excellent quality, otherwise the features with long lifetime are underweighted. Another serious problem is created by the very small, and usually, short-lived features. As they are not properly resolved, it is very difficult to identify them and determine the exact moment of their formation and their disappearance. This means that, in general, by using this method we take the risk to attribute different weights to short-lived, intermediate-lived and very long-lived features.

The second technique (method 2) consists in tagging a well-defined number of features, considered as the "initial population", on a "master" frame at the middle of a time series of photographs and tracing them back to formation and forward to disappearance. This produces a curve of the surviving features from the initial population as a function of time. The peak of this "survival" curve corresponds to the time of the "master" frame. The mean lifetime is then usually defined from this curve by using different mathematical procedures. It is evident that the application of this method is easier and more accurate for two reasons:

a) The initial population can be chosen to have a good statistical representation of all types of studied features (referring to location, size, brightness etc.).

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b) It is much easier to trace a selected feature back to its formation and forward to its disappearance instead of identifying every new born feature.

The discrepancies between the mean lifetime values given by different authors who used this second technique come from the different functions used to fit the observations, as well as from different definitions of the notion "mean lifetime". Thus, some authors define as "mean lifetime" the full width at half maximum (FWHM) of this curve (Macris and Prokakis, 1963; Janssens, 1970; Alissandrakis and Macris, 1971; Dialetis et al., 1985, 1986). In some cases the FWHM was determined after fitting of the observations with a quadratic or exponential curve. Others use an exponential fit and define as "mean lifetime" the e-folding time [see, e.g. Simon and Leighton (1964) in their measurements of the lifetime of the Ca II K network]. Others define as "mean lifetime" twice the e-folding time (see, e.g. Mehltretter, 1978 in his measurements of the lifetime of photospheric granules). All these approaches of the problem are based on a certain kind of enumeration of the features under study. It is therefore important, not only to find the most appropriate fitting curve, but also to make an accurate determination of the mean lifetime from such a curve. That is exactly the aim of this paper.

We should add that there is another approach based on the use of the autocorrelation of the intensity as a function of time. Bahng and Schwarzschild (1961) used the FWHM of an exponential fit of the autocorrelation function of the intensity with respect to time; this quantity was considered by definition as the "mean lifetime" of the granules. Such a definition of the mean lifetime is different from the "classical statistical" definition. For this reason this second approach is outside the scope of our study. However it is interesting to examine in the future the relation between measurements based on the enumeration of features and measurements based on the autocorrelation function of the intensity.

2. Formulation of the problem

In this section we will derive a relation between the lifetime distribution and the survival curve. Let us assume that at a given time (considered as $t_0 = 0$) we observe n_0 solar features. We want to know how many features of this initial sample continue to exist at any time $t > t_0$.

If we consider the curve of the surviving features from the initial population as a function of time, then at a given time t the surviving features have two properties:

a) They have lifetimes T greater than t.

b) They are born in the time interval $[t-T, 0]$.

The probability that a feature has a lifetime T is given by the probability density function of their lifetime distribution, $g(T)$, normalized to unity; assuming a uniform time distribution of births, the probability that a feature with lifetime T was born in the interval $[t-T, 0]$ is $(T-t)/T$. Consequently the combined probability that the feature will still exist at time t is $g(T)$ $(T-t)/T$. Assuming that the appearance and disappearance of two or more features are independent events, the number of surviving features, $f(t)$, is

$$f(t) = n_0 \int_t^\infty \frac{T-t}{T} g(T)\, dT, \quad t \geqq 0. \tag{1}$$

Going backward from the master frame we can consider the disappearance of a granule as its birth and vice versa and thus conclude that the survival curve is symmetric with respect to zero.

Using Eq. (1) it is possible to calculate the expected number of surviving features from an initial population as a function of time. This means that we can reproduce the curve found from observations under two assumptions:

a) We know the type of probability distribution of lifetimes of the features under study.

b) We assume, to a first approximation, that the sample is composed by features belonging to the same statistical population.

On the other hand it is also possible to calculate the probability distribution of lifetimes from the curve found from observations. Indeed, inverting Eq. (1) we obtain:

$$g(t) = \frac{t}{n_0} f'' \tag{2}$$

obviously $g(t) = 0$ for $t < 0$. Note that the requirement $g(t) > 0$ implies that $f(t)$ will be concave upwards for every value of t and its derivative will be discontinuous at $t = 0$.

The mean lifetime of the feature and its variance can be computed either from the probability density function or from the survival function:

$$\mu = \int_0^\infty t g(t)\, dt = \frac{1}{n_0} \int_{-\infty}^\infty f(t)\, dt, \tag{3}$$

$$\sigma^2 = \int_0^\infty (t-\mu)^2\, g(t)\, dt = \frac{6}{n_0} \int_0^\infty t f(t)\, dt - \mu^2. \tag{4}$$

Equations (2)–(4) show that the probability density function as well as the mean lifetime and its variance can be recovered from observations of the survival function. The mean lifetime is related to the area under the survival curve and the variance is related to its first moment. We should note however that the distribution function cannot be computed for values of time greater than one half the duration of time series and that the computation of μ will give a lower limit, unless the measurement of $f(t)$ extends up to sufficiently high values of t. In addition, the numerical computation of $g(t)$ involves the second derivative of $f(t)$, which is likely to introduce significant noise, unless the observations are fitted with a smooth curve.

3. The reconstruction of proability density function from observational curves

In this section we will compute the probability density functions which correspond to some frequently used forms for the survival function and will consider the validity of some definitions of the mean lifetime.

A usual fitting procedure is to fit the observational data with an exponential curve of the form (Fig. 1, curve A):

$$f(t) = n_0 e^{-\lambda t} \tag{5}$$

(Simon and Leighton, 1964; Janssens, 1970; Mehltretter, 1978). The corresponding probability density function of the distribution is, from Eq. (2):

$$g(t) = t\lambda^2 e^{-\lambda t}. \tag{6}$$

In that case the mean lifetime is, from Eq. (3)

$$\mu = 2/\lambda \tag{7}$$

and the standard deviation σ of lifetimes is, from Eq. (4)

$$\sigma = \sqrt{2}/\lambda. \tag{8}$$

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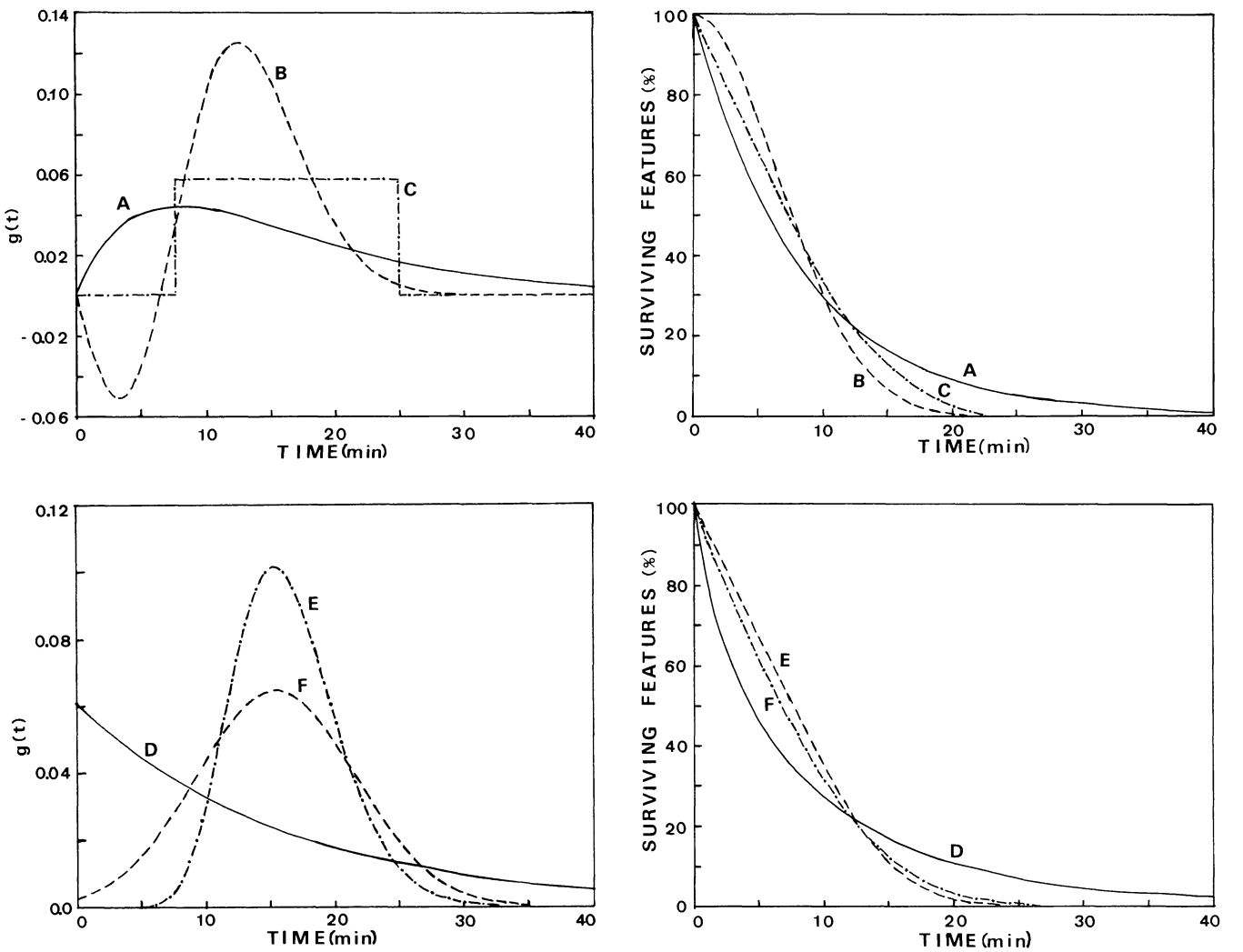

Fig. 1. Probability density functions and corresponding survival functions for six types of lifetimes distributions.
A Exponential survival function of the form $f(t) = n_0 e^{-\lambda t}$ and corresponding probability function (Sect. 3). B Survival function of the form $f(t) = n_0 e^{-\lambda^2 t^2}$ and corresponding probability function (Sect. 3). C Probability density function of a uniform distribution and corresponding survival function (Sect. 4.1). D Probability density function of an exponential distribution and corresponding survical function (Sect. 4.2). E Probability density function of a Poisson distribution and corresponding survival function (Sect. 4.3). F Probability density function of a truncated gaussian distribution and corresponding survival function (Sect. 4.4). The mean lifetime is 16.3 min for all distributions. The standard deviation is 5 min for distributions C and F; for the others the standard deviation is determined by the mean lifetime

As we can see from this example the use of a fitting function of the type (5) presupposes a probability function of the type (6). Such a density function of lifetimes is not symmetrical, it is skewed to the right (Fig. 1, A). However, statistical determinations of the probability density function of lifetimes of solar granules (Macris, 1953) does not show such a positive skewness. For this reason we believe that the use of a fitting function of this type is not the best choice, especially for the determination of the mean lifetime of the solar granulation.

In any case, from Eq. (3) it is clear that for this type of fitting function the corresponding mean lifetime μ of the sample is equal to $2/\lambda$ and not equal to $1/\lambda$ as is very often assumed. It is also clear that with a fitting function of this type it is not possible to use the FWHM for the determination of the mean lifetime, because from the FWHM $\mu = 2 \ln 2/\lambda$. That means that this technique underestimates the mean lifetime by a factor $\ln 2$ ($=0.693$).

In general it is not easy to determine a priori the type of fitting function, because such a function must not only fit the observational curve well but it must also lead to a reasonable distribution with physical meaning. For example a fitting curve of the form (Fig. 1, curve B):

$$f(t) = n_0 e^{-\lambda^2 t^2} \tag{9}$$

fits very well some observational curves, but the corresponding probability density function

$$g(t) = 2\lambda^2 t (2\lambda^2 t^2 - 1) e^{-\lambda^2 t^2} \tag{10}$$

leads to a distribution without physical meaning, since it is negative for $t < 1/(\lambda\sqrt{2})$ (Fig. 1, B).

The use of fitting function of quadratic type (Macris and Prokakis, 1963; Dialetis et al., 1985) leads also to non physical probability density function, which increases linearly with time.

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From these examples it is clear that it is more convenient to use fitting functions deduced from some well-known standard probability functions (Gaussian, Poisson, Exponential etc.) because a fitting procedure by using functions chosen with only mathematical criteria ("best fit") may give results without physical meaning.

4. The fitting function deduced from a given probability density function

In this section we give the corresponding survival functions for four standard continuous distributions (Uniform, Gaussian, Exponential, Poisson), that we consider as more appropriate for the description of the lifetimes distribution.

4.1. *Uniform distribution of lifetimes*

A very simple first case is to assume that the lifetimes have a rectangular or uniform distribution in a time interval $[\alpha, \beta]$. The corresponding density function is (Fig. 1, curve C):

$$g(t) = \begin{cases} \dfrac{1}{\beta-\alpha}, & \alpha < t < \beta \\ 0, & \text{otherwise}. \end{cases} \tag{11}$$

The mean μ and the standard deviation σ for this distribution are:

$$\mu = \frac{\beta+\alpha}{2} \quad \text{and} \quad \sigma = \frac{\beta-\alpha}{\sqrt{12}} \tag{12}$$

and the corresponding survival function is (Fig. 1, curve C):

$$f(t) = \begin{cases} n_0\left(1 - \dfrac{t}{\beta-\alpha}\ln(\beta/\alpha)\right), & 0 \leqq t \leqq \alpha \\ n_0\left(\dfrac{\beta-t}{\beta-\alpha} - \dfrac{t}{\beta-\alpha}\ln(\beta/t)\right), & \alpha \leqq t \leqq \beta \\ 0, & \beta \leqq t. \end{cases} \tag{13}$$

Such a function can be used as a first approximation in the case of a flat-topped distribution.

4.2. *Exponential distribution of lifetimes*

A more realistic approximation is to suppose that the lifetime distribution is of exponential type; then its probability density function is of the form:

$$g(t) = \lambda e^{-\lambda t}. \tag{14}$$

Such a probability density function seems reasonable only in the case of populations with a very large number of short-lived features (Fig. 1, D). The mean μ and the standard deviation σ of this distribution are

$$\mu = \sigma = \frac{1}{\lambda}. \tag{15}$$

In that case the survival function $f(t)$ from an initial sample of n_0 features, as a function of time, is given by the equation

$$f(t) = n_0\,[e^{-\lambda t} - \lambda t E_1(\lambda t)], \tag{16}$$

where $E_1(\lambda t)$ is the exponential integral (Fig. 1, curve D).

4.3. *Poisson distribution of lifetimes*

In some cases, the distribution of lifetimes can be approximated by a continuous Poisson distribution. In that case the probability density function $g(t)$ is given by:

$$g(t) = \frac{1}{\Gamma(\alpha+1)}\, t^{\alpha} e^{-t}, \tag{17}$$

where $\Gamma(\alpha+1)$ is the well-known gamma-function (Fig. 1, curve E).

The mean μ and the standard deviation σ of this distribution are:

$$\mu = \alpha + 1 \quad \text{and} \quad \sigma = \sqrt{\alpha+1}\,. \tag{18}$$

In that case the survival function $f(t)$ is of the form (Fig. 1, E):

$$f(t) = n_0 \frac{1}{\Gamma(\alpha+1)}\,[(\alpha - t)\,\Gamma(\alpha, t) + t^{\alpha} e^{-t}], \tag{19}$$

where $\Gamma(\alpha, t)$ is the incomplete gamma-function.

4.4. *Gaussian distribution of lifetimes*

The most probable distribution for the lifetimes of solar features is a Gaussian distribution, with probability density function $g(t)$ given by

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right] \tag{20}$$

for $t > 0$ (truncated Gaussian), where μ is the mean lifetime and σ the standard deviation of lifetimes (Fig. 1, curve F).

In that case the survival function $f(t)$ for an initial sample n_0, is given by the equation

$$f(t) = n_0\left[Q(s) - t\int_s^{\infty} \frac{1}{\sqrt{2\pi}}\,\frac{1}{\sigma y + \mu} \exp\left(-\frac{y^2}{2}\right) dy\right], \tag{21}$$

where $s = (t-\mu)/\sigma$ and $Q(s)$ is the error function (Fig. 1, F).

From the form of all these fitting functions it is clear that for the corresponding standard distributions the FWHM underestimates the mean lifetime.

5. The determination of the mean lifetimes of solar granules

A very good example of the discrepancies produced essentially by the different methods used for the mean lifetimes determination of solar features, is the case of the solar granules.

We dispose a large number of mean lifetime determinations of the solar granules, made by previous authors by using different methods and different fitting functions. The first adequate determination of the lifetime of solar granulation was made by Macris (1953), who estimated the lifetimes of individual granules and found $\approx$7.5 min. Later Bray and Loughhead (1958) obtained results in agreement with those of Macris ($>$7–8 min). Rosch and Hugon (1959) gave 10 min, Macris and Prokakis (1953) about 8 min, Simon (1967) 8 min, LaBonte et al. (1975) $\approx$15 min and Mehltretter (1978), 16 min. Recently Dialetis et al. (1986) gave $\approx$12 min for mean lifetime.

It is interesting to consider whether the observed differences are real or they are due to the different methods used. If the

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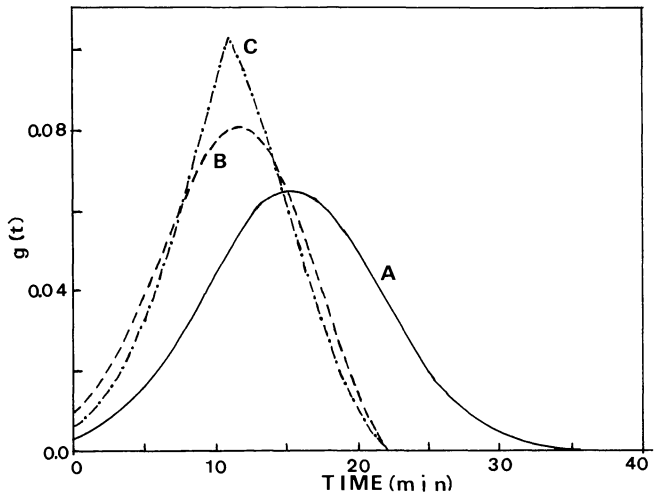

Fig. 2. Effect of the limitation of the observing time on the observed distribution of the lifetimes for a gaussian parent population with mean = 16.3 min and standard deviation = 5 min *A*. *B* shows the distribution obtained by the method 1a for an observing interval of 22 min. *C* shows the distribution obtained by the method 1b for an observing interval of 22 min

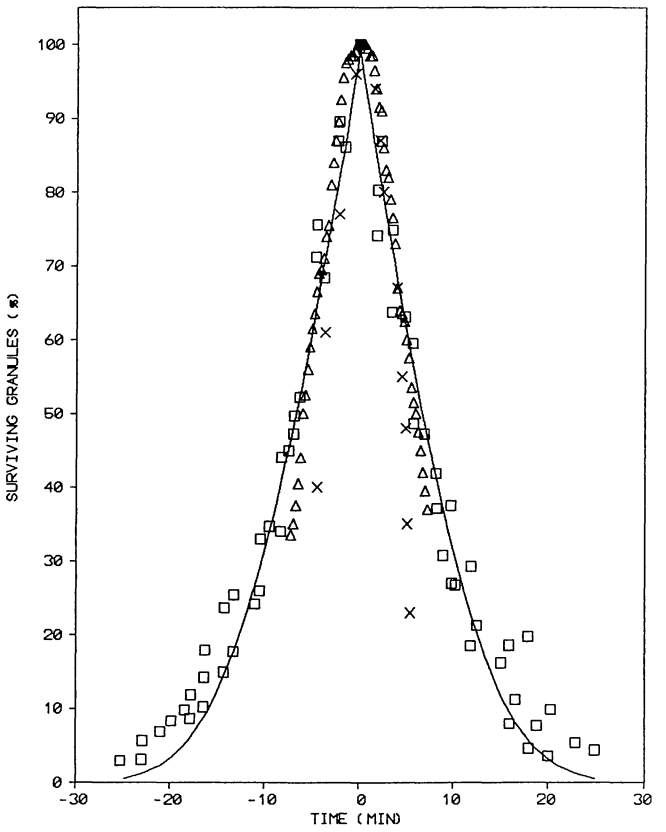

Fig. 3. Survival curves of Macris and Prokakis (1963) (crosses), Mehltretter (1978) (squares) and Dialetis et al. (1986) (triangles) and a fit of the combined survival curve (solid line) obtained for a truncated gaussian lifetime distribution ($\mu = 16.3$ min, $\sigma = 5$ min)

observed differences are real, we may assume a possible relation of the mean lifetime with the solar cycle or with the location of the sample (see also Dialetis et al., 1986). For this reason we assume that the present results can be applied in the case of solar granulation. This is true only as a first approximation because some granules are born from the splitting or the merging of other granules, which means that the disappearance and the birth of granules are not totally independent events. We also assume that all the granules of the sample belong to the same statistical population. This assumption presupposes that the location of a granule, near a strong magnetic field does not change its lifetime.

A first point that we should make is that the earlier determinations of the mean lifetime are based on relatively short duration sequences of photographs. This inconvenience, in combination with the method used (methods 1a and 1b) leads to a serious underestimation of the mean lifetime (Fig. 2), because the starting and the ending times of the long-lived granules fall outside the period of observation. The problem was pointed out by Bray and Loughhead (1958) who used a 10 min sequence and gave the calculated mean lifetime of 8 min as a lower limit of the true mean lifetime, but their evaluation of the unterestimation of the method was very optimistic. Later Mehltretter (1978) proposed a method of correction based on the assumption of a uniform distribution of births in a given time interval.

The numerical example that we give (Fig. 2) shows that the underestimations of these earlier calculations based on the determination of the lifetime of every new emerging granule, can be up to $\approx 50\%$; for this reason we must consider the given values only as a lower limit of the mean lifetime.

Another problem with these earlier determinations of the mean lifetime is due to the different criteria adopted for the 'birth" and the "disappearance" of a granule. These criteria are subjective and consequently we cannot evaluate their effect.

Simon (1967) was the first to use a method based on the survival function in combination with an exceptional sequence obtained at the Sacramento Peak Observatory. For the determination of the mean lifetime Simon used an exponential fitting function (see Sect. 3), but the *e*-folding time which he considered as mean lifetime, is, as we have shown, one half of the mean lifetime of the assumed distribution. In view of that, his results are in good agreement with those of Mehltretter (1978) who gave a value of 16 min for twice the *e*-folding time. Very similar results ($\mu = 15$ min) were obtained by LaBonte et al. (1975).

In an attempt to further reconcile the previous results we re-analyzed the survival curves of Macris and Prokakis (1963), Mehltretter (1978) and Dialetis et al. (1986), for which we disposed numerical values. Their measurements are shown in Fig. 3, together with a fit of the combined survival curve. Among the curves considered in Sect. 3 and 4, the best fit was obtained for a truncated gaussian lifetime distribution with average lifetime of 16.3 min and standard deviation of 5 min. The values of Macris and Prokakis (1963) are in good agreement with the fit for short values of t, however they fall below the curve for larger values, probably because of the lower quality of their photographs. The values of Dialetis et al. are in very good agreement with the fit but the mean lifetime that they gave (≈ 12.5 min) by using the FWHM method is underestimated by a factor ≈ 0.75 as it was expected from the previous analysis (see Sect. 3 and Fig. 1, F).

6. Conclusions

In the previous sections we have unified two approaches that we commonly used in the determination of the mean lifetime of solar features; the approach based on the lifetime distribution and that

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based on the survival function. We give the relation between the survival function $f(t)$ and the probability density function $g(t)$ of the corresponding distribution, as well as the relations between the survival function and the mean lifetime and the standard deviation of the parent population. These relations give the possibility to obtain with relatively simple computational techniques the values of mean lifetime and standard deviation directly from the survival function. The accuracy of the computed values is a function of the duration of the observation.

We have shown that in some cases the functions used for fitting the survival function correspond to distributions without statistical or physical meaning. For this reason we have given survival functions that correspond to standard distributions suitable for the modeling of lifetime distributions. By means of these functions we have shown that there is no relation between the mean lifetime and the value obtained from the FWHM. For the cases of standard distributions that we have studied the FWHM clearly underestimates the mean lifetime.

Finally we have examined the results given from previous authors for the case of solar granulation and we have shown that the discrepancies between these results are mainly due to the methods used. We conclude that the best value for the mean lifetime is $\approx$16 min and not 6–9 min, as is actually mentioned in the literature (Bray et al., 1984). As for the type of distribution our calculations, based essentially on the measurements given by Mehltretter (1978) and Dialetis et al. (1986), suggest that it is more likely a truncated Gaussian distribution with $\mu \approx 16$ min and $\sigma \approx 5$ min.

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